Analysis of non-quarter-wave grating by a modified Fourier-transform method

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When unity reflectance is approached, the Fourier-transform method of calculating the reflectance spectrum of an optical grating modulated by a slowly varying envelope becomes unacceptably inaccurate. The modified Fourier transform method of Bovard [Appl. Opt. **29,** 24 (1990)] can achieve complete accuracy for quarter-wave gratings. We report herein the extension of Bovard's method to non-quarter-wave gratings. We demonstrate the accurate deployment of our simplified modified Fourier-transform method to apodized linear gratings and optically apodized nonlinear gratings. © 2002 Optical Society of America

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1. Introduction

The synthesis of filters is widely and well understood through Fourier-transform (FT) methods. An entire body of convenient computational methods, as well as a shared intuition and vocabulary, surrounds the use of methods such as apodization to engineer the spectrum of a filter.

This is true in some respect for optical filters, but not fully. The FT relation between the reflectance and the refractive-index profile of a grating[1–10] becomes particularly inexact as the reflectance approaches unity, a regime of central interest in, for example, wavelength-selective add–drop filters for multiwavelength networks. The modified Fourier-transform method (MFT) of Bovard[1] is accurate even when reflectance is close to 100%; however, it works only with a quarter-wave structure.

2. Review of the Method of Bovard for the Special Case of Quarter-Wave Stacks

The quarter-wave stack[11,12] is specified in Fig. 1, with $a = b$. The logarithmic derivative $r(x)$ of the refractive-index profile is

$$r(x) = r_0 \sum_{m=-p}^{p-1} \left[\delta\left(x - \frac{4m+3}{4\sigma_0}\right) - \delta\left(x - \frac{4m+1}{4\sigma_0}\right) \right], \quad (1)$$

where x is the centered double optical thickness, $p = N/2$, $r_0 = [\ln(n_h/n_l)]/2$, and $\sigma_0 = 1/\lambda_0$. The logarithmic derivative consists only of components that are due to discontinuous interfaces in the refractive-index profile.

The (MFT) relationship between logarithmic derivative $r(x)$ and reflectance spectral function Q derived by Bovard is[1,3]

$$Q(\nu)\exp[i\Phi(\nu)] = \int_{-\infty}^{+\infty} r(x)\exp(-2i\pi v x)\mathrm{d}x, \quad (2)$$

where v is the wave-number distortion factor:

$$v = \frac{2\sigma_0}{\pi}\cosh^{-1}\left(\cosh r_0 \sin\frac{\pi\sigma}{2\sigma_0}\right), \quad (3)$$

$\sigma = 1/\lambda$, λ is the wavelength, and $r(x)$ is the logarithmic derivative $[\mathrm{d}n(x)/\mathrm{d}(x)]/2n(x)$. $Q(v)$ is a spectral function related to the reflectance. Because we are interested in reflectance R, we require Eq. (2) to be rewritten as

$$|Q(\nu)| = \left| \int_{-\infty}^{+\infty} r(x)\exp(-2i\pi\nu x)\mathrm{d}x \right|. \quad (4)$$

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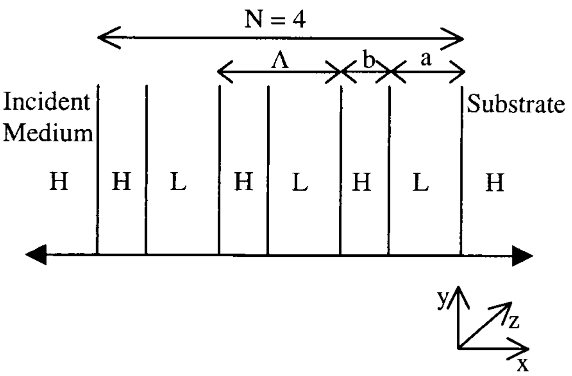

Fig. 1. Periodic stack consisting of N periods of alternating layers H with high index, n_h and L with low index, n_l. The incident and substrate media are assumed to have refractive index Hn_h. x is the centered double optical thickness. The special case $a = b$ describes the quarter-wave stack.

Substituting Eq. (1) into Eq. (4) yields the following value for Q:

$$|Q(\nu)| = \left| r_0 \sum_{m=-p}^{p-1} \left\{ \exp\left[-2i\pi \frac{(4m+3)v}{4\sigma_0} \right] - \exp\left[-2i\pi \frac{(4m+1)v}{4\sigma_0} \right] \right\} \right|. \tag{5}$$

After obtaining $|Q(\nu)|$ from Eq. (5), Bovard[1] proposed to calculate the reflectance differently for two cases:

Case 1: Inside the reflection band [when v is complex in Eq. (3)], the expression for reflectance is

$$R = \left[\frac{\exp(2|Q|) - 1}{\exp(2|Q|) + 1} \right]^2. \tag{6}$$

Case 2: Outside the reflection band [when v is real in Eq. (3)], the expression for reflectance is

$$R = \frac{|Q|^2}{1 + |Q|^2}. \tag{7}$$

In the reflectance calculations above it is assumed that the material used is nonabsorbing, such that the sum of transmittance and reflectance is equal to unity. The result is exact for the special case of the quarter-wave stack.[1]

3. Extending the Method of Bovard to the Special Case of Non-Quarter-Wave Stacks

We now show how both the traditional FT method[2] and also the MFT of Bovard may be extended to predict accurately the behavior of a non-quarter-wave stack. We find that the MFT method is more accurate than the traditional FT method for non-quarter-wave stacks and that the MFT method is reasonably accurate compared with the exact solution provided by the transfer matrix multiplication[12] (TMM) method.

The stack again consists of two layers, H and L, of

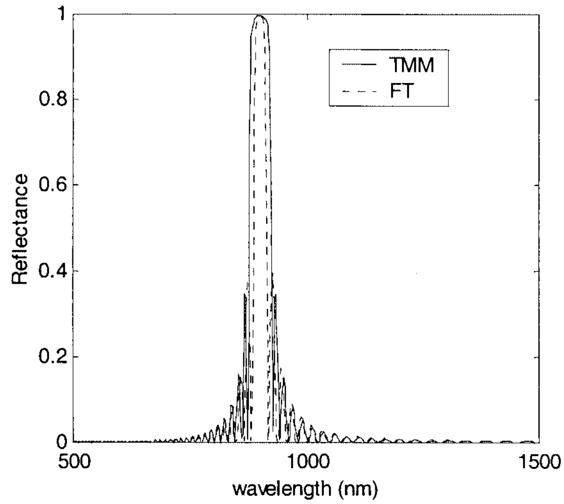

Fig. 2. Comparison of the reflectance of a non-quarter-wave stack as computed by the TMM and the FT methods. The layers have thicknesses $a = 250$ nm and $b = 200$ nm, with $N = 50$, $n_h = 1.5$, and $n_l = 1.4$.

optical thicknesses a and b. The logarithmic derivative $r(x)$ becomes

$$r(x) = r_0 \sum_{m=-p}^{p-1} (\delta\{x - [2a + b + 2m(a+b)]\} - \delta\{x - [b + 2m(a+b)]\}). \tag{8}$$

We use the traditional FT method for $r(x)$ of Eq. (8) to obtain the reflectance plotted in Fig. 2. Agreement is poor; for example, the stop-band width predicted deviates from the exact value from TMM method by 40%.

Encouraged by this result, we proceed to derive the extension of MFT to the non-quarter-wave stack case. For gratings that involve two different optical thicknesses, the Bragg condition is given by[12]

$$a + b = \lambda_1/2, \tag{9}$$

where a and b are the optical thicknesses of the two layers, and λ_1 is the reference wave that has the highest reflectance. We set up an equivalent grating system with equal optical thickness such that

$$a_1 = b_1 = \lambda_1/4. \tag{10}$$

Once we convert the non-quarter-wave stack to an equivalent quarter-wave stack, using Eqs. (9) and (10), we may employ wave-number correction factor v in Eq. (3) to calculate the MFT on the equivalent quarter-wave stack with $\lambda_1/4$ thickness. Substituting Eq. (8) into Eq. (4) yields

$$|Q(\nu)| = \left| r_0 \sum_{m=-p}^{p-1} (\exp\{-2i\pi\nu[2a + b + 2m(a+b)]\} - \exp\{-2i\pi\nu[b + 2m(a+b)]\}) \right|. \tag{11}$$

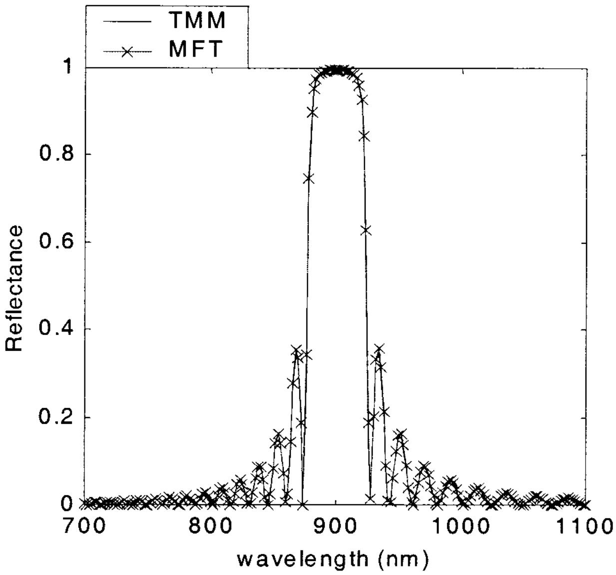

Fig. 3. Comparison of the reflectance of a non-quarter-wave stack as computed by the TMM and the MFT methods. The layers have thicknesses $b = 200$ nm and $a = 250$ nm, with $N = 50$, $n_h = 1.5$, and $n_l = 1.4$.

We illustrate in Fig. 3 the result for $a = \lambda_0/4$, $b = \lambda_0/5$, and $\lambda_0 = 1000$ nm. The exact and MFT results agree within 5%. The match degrades as the difference between a and b becomes larger. For $a = 250$ nm and $b = 100$ nm, the reflectance computed by the MFT on an equivalent quarter-wave stack and by TMM method exhibits a discrepancy of 25%, as shown in Fig. 4.

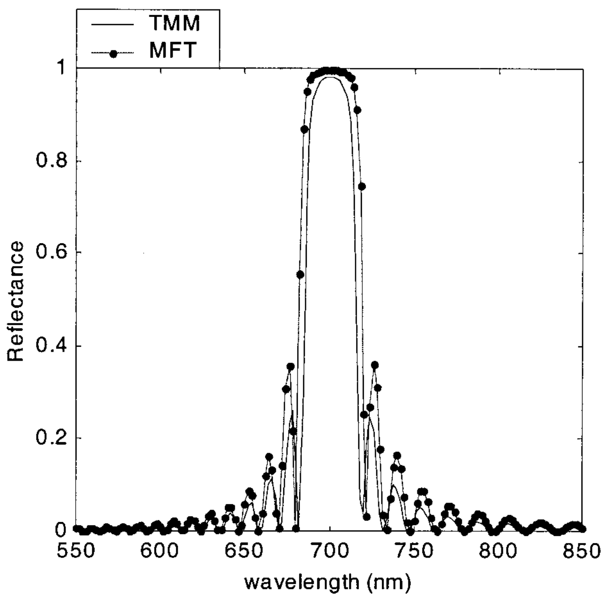

Fig. 4. Comparison of the reflectance of a non-quarter-wave stack as computed by the TMM and the MFT methods. The layers have thicknesses $b = 100$ nm and $a = 250$ nm, with $N = 50$, $n_h = 1.5$, and $n_l = 1.4$.

4. Application of a Simplified Modified Fourier-Transform Method to an Apodized Bragg Grating

We now apply the MFT method to an apodized grating. If we denote the apodization envelope $E(x)$, and the quarter-wave stack's refractive-index profile

$$n(x) = C(x) + \bar{n}, \tag{12}$$

where $C(x)$ is the ac component and $\bar{n}$ is the dc component, which is the average refractive index, the apodized gratings become

$$N_{\text{apod}}(x) = E(x)C(x) + \bar{n}. \tag{13}$$

We derived the corresponding logarithmic derivative profile $r(x)$ as follows:

$$r_{\text{apod}}(x) = \tfrac{1}{2}[\ln N_{\text{apod}}(x)]' = \tfrac{1}{2}\left[\frac{E'(x)C(x) + E(x)C'(x)}{E(x)C(x) + \bar{n}}\right].$$

[since $|E(x)C(x)| \ll \bar{n}$]

$$\approx \tfrac{1}{2}\left[\frac{E'(x)C(x)}{\bar{n}} + \frac{E(x)C'(x)}{\bar{n}}\right],$$

[slowly varying envelope $E(x)$]

$$\approx \tfrac{1}{2}\left[\frac{E(x)C'(x)}{\bar{n}}\right]. \tag{14}$$

Expression (14) is of the form $r(x) = e(x)c(x)$—a carrier modulated by an envelope function—where $c(x) = C'(x)/2\bar{n}$ and $e(x) = E(x)c(x)$ is the logarithmic derivative of the quarter-wave stack as defined in Eq. (1). To calculate the reflectance of apodized gratings we can simply compute the MFT of $r_{\text{apod}}(x) = c(x)E(x) \leftrightarrow Q_{\text{apod}}(\nu)$ to obtain the reflectance spectra by using Eqs. (6) and (7).

Results of this simplified MFT (SMFT) method are given in Fig. 5, which shows a comparison of reflectance spectra computed by the SMFT method, the traditional FT method, and the recursion method on four kinds of envelope, i.e., Hanning, Hamming, Gaussian, and Kaiser windows. In all cases the SMFT method is more accurate than the traditional FT method and approaches closely (within 10%) the exact solution.

5. Simplified Modified Fourier-Transform on Nonlinear Gratings

Similarly to apodized gratings, nonlinear gratings can be treated as apodized gratings whose envelopes change with the local intensity of light within the structure. We consider a nonlinear optical structure proposed by Brzozowski,[13] which is similar to the structure shown schematically in Fig. 1, consisting of alternating nonlinear materials that have opposite-signed Kerr coefficients but equal linear indices n_0. The index of refraction can be expressed as[14]

$$n = n_0 + n_{\text{nl}}I, \tag{15}$$

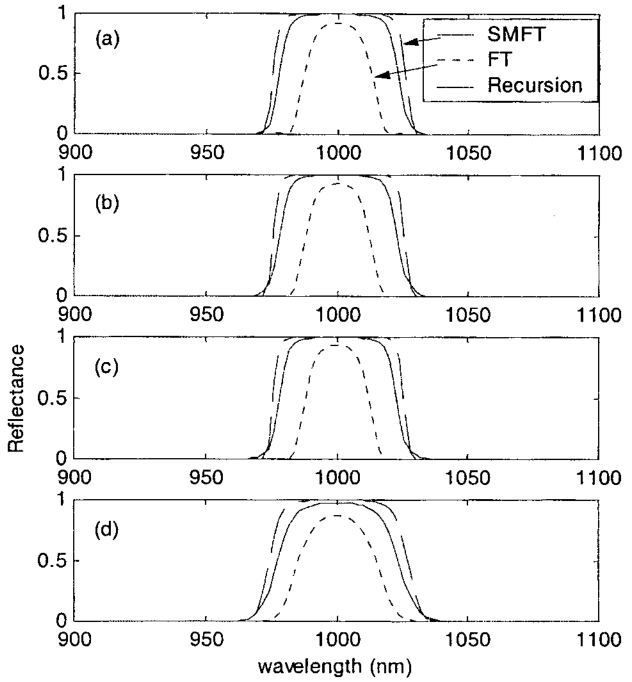

Fig. 5. Comparison of apodized-grating reflectance calculated by use of the simplified MFT, FT, and recursion (exact) methods. (a) Hanning window, (b) Hamming window, (c) Kaiser (β = 5), window, and (d) Gaussian window.

where n_0 is the linear part, $n_{\rm nl}$ is the Kerr coefficient, and I is the intensity of light in the medium. The layers possess $|n_{\rm nl}|$ of the same magnitude but with opposite sign, and the intensity-dependent index of refraction decreases or increases with intensity. The average index, and therefore the spectral position of the center of the stop band, remains fixed. Only the width of the stop band varies. A control beam $I_{\rm control}$ may be used to influence the refractive indexes such that a low-intensity signal beam $I_{\rm signal}$ will experience different reflectance spectra.

We choose $I_{\rm control}$ to have a frequency that is resonant with the structural periodicity to achieve a high degree of modulation.[13] The intensity within the structure is given by the expression

$$I(x) = 2I_1(x) - I_{\rm out}, \qquad (16)$$

where I_1 denotes the forward-propagating wave and is given by[14]

$$I_1(x) = \left| \frac{1 + \cos \dfrac{4I_{\rm out}n_{\rm nl}(L-x)}{\Lambda n_0}}{2\cos \dfrac{4I_{\rm out}n_{\rm nl}(L-x)}{\Lambda n_0}} \right| I_{\rm out} \qquad (17)$$

and where L is the total length of the structure.

The value of $I_{\rm control}$ is given by

$$I_{\rm control} = I_1(0) = \frac{1}{2} \left| \frac{1}{\cos\left(\dfrac{4I_{\rm out}}{a}\right)} + 1 \right| I_{\rm out}, \qquad (18)$$

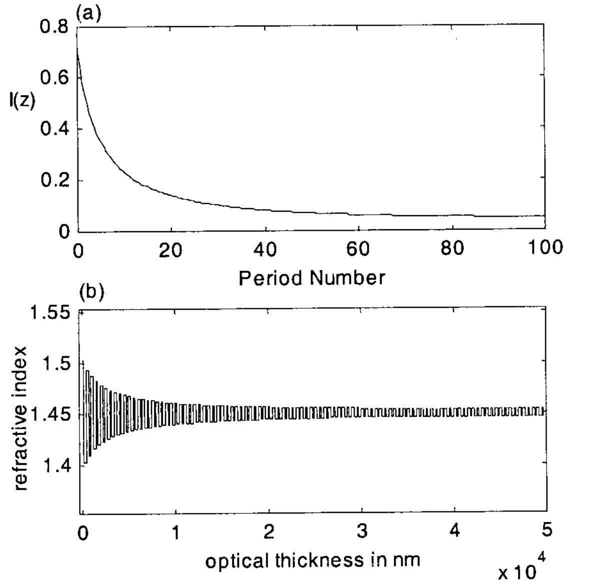

Fig. 6. (a) Intensity profile $I(z)$ inside the nonlinear structure with $n_0 = 1.45$, $n_{\rm nl} = 0.075$, and $I_{\rm out} = 0.05$ (top); and (b) $N = 100$ periods and the corresponding refractive-index profile.

where $a = n_0/Nn_{\rm nl}$ and $N = L/\Lambda$ is the number of periods.

By choosing Λ, n_0, $n_{\rm nl}$, and $I_{\rm out}$ in Eq. (17) and substituting them into Eq. (16) we obtain the intensity profile across the structure. To apply the SMFT method we treat $I(x)$ in Eq. (16) as an envelope function that slowly modulates the rapidly oscillating component $\pm n_{\rm nl}$. This envelope function $I(x)$ is shown in Fig. 6. For comparison, the reflectance spectra for the nonlinear structure and a linear (unapo-

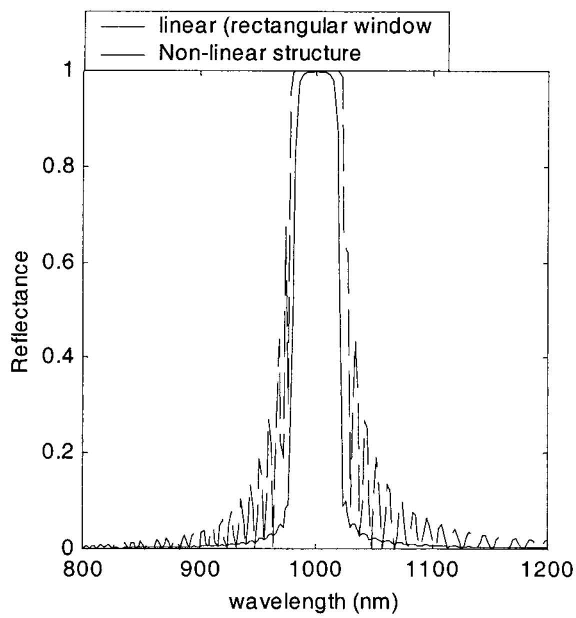

Fig. 7. Reflectance of the nonlinear grating of Fig. 6 and of a linear grating (quarter-wave) that has $n_0 = 1.45$ and index difference $\Delta = 0.15$ as calculated with the MFT method.

dized) structure are shown in Fig. 7. Apodization caused by the decaying envelope in the nonlinear case smooths the spectrum.

Finally, we note that other grating profiles such as a Gaussian-like envelope can be synthesized by use of an orthogonal control beam on the z or the y axis—one whose profile is not influenced by the nonlinear grating but that may instead be engineered deterministically in transverse space.

6. Conclusions

We have introduced a simplified modified Fourier-transform method for calculating the reflectance spectra of an apodized grating and a non-quarter-wave grating. We have shown the increased accuracy of the SMFT in the calculation of reflectance in both linear and nonlinear gratings.

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