# System Performance Comparison of Optical CDMA and WDMA in a Broadcast Local Area Network 

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#### Abstract

The performance of optical code-division multipleaccess (CDMA) systems with wavelength-hopping/time-spreading codes is compared to that of a wavelength-division multiple-access (WDMA) system. The multiple-access techniques are applied in a time-slotted broadcast local area network. The utilization, defined as the throughput per unit of time-domain bandwidth expansion, and packet delay are used as metrics of performance. When more than seven wavelengths are available, optical CDMA systems using asymmetric prime-hop codes and all-optical signal processing are shown to have higher peak utilization and lower corresponding delay than a WDMA system with the same number of wavelengths. When the encoders/decoders operate at the chip rate, the utilization of optical CDMA exceeds that of WDMA at high offered loads; however, the peak utilization of the WDMA system is still superior.


Index Terms-Code-division multiaccess, optical communication, optical fiber LAN, wavelength division multiplexing.

## I. Introduction

OPTICAL code-division multiple access (CDMA) is a method of sharing the bandwidth of optical fiber among a number of active users in a broadcast local area network [1], [2]. There are numerous advantages in using 2-D (wavelength-time) codes [3] in these optical CDMA systems. In addition to having favorable cross-correlation and autocorrelation characteristics, two-dimensional codes allow a large number of users to be supported with much less time-spreading than a pure time-domain (1-D) code having the same cardinality.

Given that each station in an optical CDMA network with 2-D codes can produce a number of wavelengths, wavelength-division multiple access (WDMA) could be implemented as an alternative without a major hardware upgrade. Optical CDMA has the advantage that the number of active users can be much larger than the number of available wavelengths; however, the cost is bandwidth expansion and a nonzero bit-error rate (even when all physical noise is neglected) due to interference between the users. WDMA does not require bandwidth expansion and does not suffer from multiple-access interference but the number of simultaneous users is limited by the number of available wavelengths. In the analysis that follows, the system performance of 2-D optical CDMA is compared to the performance of a WDMA system with the same number of wavelengths.

Typically, the transmitter and receiver structures of an optical CDMA station employ optical components to perform high-

[^0]speed signal processing. Optical CDMA encoders built with tunable Bragg gratings [4] or tapped delay lines [1] have been demonstrated. These optical encoders allow the electronics of each station to operate at the user data rate, rather than the chip rate; however, if the stations implement either a passive-correlator or a high-speed chip-level detector, the receiver electronics must operate at the chip rate [5]. Therefore, in this manuscript we consider optical CDMA systems that operate both at the data rate (denoted all-optical CDMA) and at the chip rate (termed chip-rate CDMA).

## II. Analytical Framework

We consider the synchronous, random-access, packet broadcast network described by Raychaudhuri [6]. Users begin transmissions on common clock instances and the length of a slot corresponds to a packet of length $L$ bits. With a suitable mul-tiple-channel multiple-access scheme (WDMA or CDMA), a number of packets from different sources can be transmitted over the optical fiber in a single slot. We denote the number of simultaneous packets on the channel during a slot interval by $m$.

Due to multiple-access interference, some of the packets will arrive at the receiver with bit errors. We let $P_{B}(m)$ be the probability of a bit error when there are $m$ simultaneous transmissions on the channel. The form of $P_{B}(m)$ will depend upon the type and parameters of the specific multiple-access scheme. The probability of receiving a packet without errors when $m$ simultaneous transmissions are on the channel is given by

$$
\begin{equation*}
P_{C}(m)=\left[1-P_{B}(m)\right]^{L} \tag{1}
\end{equation*}
$$

With suitable error-detection capability, the receiver can determine if one or more errors have occurred in a packet. All packets with errors are dropped by the receiver. For simplicity, we neglect the overhead required for this error-detection. In a broadcast network, the sender can independently determine the success or failure of the transmission and schedule the packet for retransmission after a random delay.

Let $M$ be a random variable that represents the number of simultaneous transmissions in a time slot. The conditional distribution of the number of successfully received packets $S$ is then

$$
\begin{equation*}
P[S=s \mid M=m]=\binom{m}{s} P_{C}^{s}(m)\left[1-P_{C}(m)\right]^{m-s} \tag{2}
\end{equation*}
$$

The steady-state throughput $\beta$ can be shown [6] to equal

$$
\begin{equation*}
\beta=E[S]=E[E[S \mid M]]=\sum_{m=1}^{\infty} m P_{C}(m) f_{M}(m) \tag{3}
\end{equation*}
$$

where $f_{M}(m)$ is the steady-state probability distribution of composite arrivals (new and retransmitted packets).

We assume that the composite arrival distribution is Poissonian with arrival rate $\lambda$

$$
\begin{equation*}
f_{M}(m)=\frac{(\lambda T)^{m}}{m!} e^{-\lambda T} \tag{4}
\end{equation*}
$$

This choice of arrival distribution corresponds to an infinite user population. Defining $\gamma \triangleq \lambda T$ to be the offered load (average number of attempted transmissions per time slot), the throughput becomes

$$
\begin{equation*}
\beta=e^{-\gamma} \sum_{m=1}^{\infty} m P_{C}(m) \frac{\gamma^{m}}{m!} \tag{5}
\end{equation*}
$$

The delay, measured in the average number of retransmissions per packet, can be shown in this case [6] to equal

$$
\begin{equation*}
d=\frac{\gamma}{\beta-1} . \tag{6}
\end{equation*}
$$

## III. Multiple-Access Schemes

We now have a framework that describes a simple, general, local-area network. The characteristics of both optical CDMA and WDMA can be represented through the error-free packet probability $P_{C}(m)$.

## A. Optical CDMA

We confine our analysis to optical CDMA systems that employ the asymmetric prime-hop sequences described by Tančevski et al. [3]. The 2-D codes in this family have low cross-correlations, nonexistent autocorrelation sidelobes and large cardinalities. The performance of different code families has not been considered in this letter; however, the sparseness (low ratio of lit to unlit chips) of the asymmetric prime-hop sequences is typical of many other optical CDMA code families. The authors therefore believe that the general trends in the results of this letter should apply well to other two-dimensional optical CDMA codes.

The bit-error rate of asymmetrical prime-hop sequences is given by [3]

$$
\begin{align*}
& P_{B}(m)=\frac{1}{2} \sum_{i=1}^{m-1}\binom{m-1}{i} 2^{-(m-1)} \\
& \cdot \sum_{j=\theta}^{i}\binom{i}{j}\left(\frac{\left\langle\mu_{\lambda}\right\rangle}{p_{s}^{2}}\right)\left(1-\frac{\left\langle\mu_{\lambda}\right\rangle}{p_{s}^{2}}\right)^{i-j} \tag{7}
\end{align*}
$$

where $p_{s}$ (prime) is the number of time pulses per wavelength, $p_{h}>p_{s}$ (prime) is the number of available wavelengths, $\theta=p_{s}$ is the threshold and

$$
\begin{array}{r}
\left\langle\mu_{\lambda}\right\rangle=\frac{1}{\binom{p_{h}}{p_{s}}}\left[\binom{p_{h}-1}{p_{s}-1} \frac{\left(p_{s}-1\right)\left(p_{s}-2\right)+\left(p_{h}-2\right)}{p_{h}-2}\right. \\
\left.+\binom{p_{h}-1}{p s} \frac{p_{s}\left(p_{s}-1\right)}{p_{h}-2}\right] \tag{8}
\end{array}
$$

is the average number of wavelengths common to any pair of asymmetric prime-hop sequences.

In deriving this error probability, all sources of physical noise were neglected; only the impact of multiple-access interference
on the bit-error rate was included. This is a good approximation when the signal-to-noise ratio is large; however, in a power-limited regime, all-optical and chip-rate receivers structures have different BER performance [5]. Therefore, the results presented below could be considerably altered in the power-limited case.

The asymmetric prime-hop sequences have a length in the time domain of $p_{s}^{2}$ chips and can support a maximum of $p_{s}\left(p_{h}-1\right)$ simultaneous users. Without optical signal processing, an optical CDMA transmitter using a 2-D asymmetric prime-hop sequence requires an electronic processing speed of $p_{s}^{2}$ times the per-user data rate.

In terms of the analytical framework discussed in Section II, the probability of receiving a correct packet is

$$
P_{C}(m)= \begin{cases}{\left[1-P_{B}(m)\right]^{L},} & 0 \leq m \leq p_{s}\left(p_{h}-1\right)  \tag{9}\\ 0, & p_{s}\left(p_{h}-1\right)<m<\infty\end{cases}
$$

The probability given in (9) represents an upper bound; there is the implicit assumption that no two users transmit with the same code during the same slot.

## B. Wavelength-Division Multiple Access (WDMA)

For the WDMA system, we neglect all physical noise and channel impairments. With $p_{h}$ available wavelengths, the probability of receiving a correct bit is

$$
P_{C}(m)= \begin{cases}1, & 0 \leq m \leq p_{h}  \tag{10}\\ 0, & p_{h}<m<\infty\end{cases}
$$

Again, the probability given in (10) is an upper bound since we neglect the fact that two or more users may choose to transmit on the same wavelength during a slot. Accounting for the influence of more complex medium-access control protocols (with reservation or collision-detection, for example) on the error probabilities (9) and (10) is an area that merits further investigation.

## IV. Results

We must account for the time-domain bandwidth expansion of the chip-rate optical CDMA system if we wish to make a fair performance comparison. We follow Raychaudhuri's [6] lead and normalize the throughput with respect to the degree of electronic temporal spreading in order to compare dissimilar random-access systems. We define the utilization

$$
\begin{equation*}
U=\frac{\beta}{B} \tag{11}
\end{equation*}
$$

as the ratio of the throughput $\beta$ to the electronic time-domain bandwidth expansion $B$. For the WDMA system and all-optical CDMA system we have $B=1$ while for the chip-rate CDMA system with asymmetric prime-hop codes $B=p_{s}^{2}$.

Fig. 1 shows the plots of utilization versus offered load for both the WDMA and optical CDMA systems with $p_{h}=17$ available wavelengths. The corresponding curves of delay versus utilization are given in Fig. 2. For the optical CDMA systems, various degrees of time-domain encoding (in the range $3 \leq p_{s} \leq 13$ ) are plotted.

With more time-domain encoding, the peak utilization of the all-optical and chip-rate CDMA systems increased. For both, the additional coding overhead allowed a larger number of simultaneous users to be supported, increasing the utilization.


Fig. 1. Utilization versus offered load for WDMA and CDMA systems with $p_{h}=17$ wavelengths and packets of length $L=1024$ bits. The length of the optical CDMA codes in the time domain is indicated.


Fig. 2. Delay versus utilization for WDMA and CDMA systems with $p_{h}=$ 17 wavelengths and packets of length $L=1024 \mathrm{~b}$. The length of the optical CDMA codes in the time domain is indicated.

When the amount of time-domain coding was large, the all-optical CDMA system had a higher utilization over a wide range of offered loads compared to the WDMA system. Further, Fig. 1 illustrates that WDMA had a higher utilization at low offered loads compared to the chip-rate optical CDMA. When the degree of time-encoding was high, the performance of chip-rate optical CDMA actually exceeded that of WDMA at high offered loads.

Fig. 3 shows a plot of the peak utilization as a function of the number of wavelengths. We observe two distinct regimes. With less than approximately seven wavelengths, the WDMA system has a higher peak utilization than both the all-optical and chip-rate CDMA systems. With more than seven wavelengths, the all-optical CDMA system always has the highest peak utilization. In this regime, WDMA has a higher peak utilization


Fig. 3. Peak utilization versus the number of wavelengths for both optical CDMA and WDMA. The lines are only intended to help guide the eye.
than the chip-rate optical CDMA; however, for a larger number of wavelengths, the performance gap between the WDMA and chip-rate CDMA systems narrows. As we have seen in the previous section, the chip-rate CDMA can still have a higher utilization at large offered loads.

## V. Conclusion

Both all-optical and chip-rate optical CDMA systems were shown to have a higher utilization than WDMA in certain regimes of operation. The results suggest that optical CDMA, both all-optical and chip-rate, may have an important place in future optical local-area networks. More research is needed to determine the networking niche in which each technology would be most suitable.

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