An Accurate Model for Evaluating Blocking Probabilities in Multi-Class OBS Systems

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***Abstract*—This letter presents an analytical model that can be used to evaluate the blocking probability of each service class in a multi-class optical burst-switching network. The model allows, for the first time, the evaluation of blocking probabilities in optical burst switching (OBS) systems with arbitrary burst-length distributions and arbitrary offsets. This includes OBS systems in which the mean burst length of each class is different. Such systems do not follow the conservation law and cannot, therefore, be analyzed using previously published OBS models. For an OBS system with two classes, an offered load of 10^{-3}, and a 1:5 ratio of high-priority to low-priority traffic, we show that our model accurately predicts the blocking probability for each class, whereas predictions from previously published models that assume conservation disagree with the simulation results by as much as 75%.**

***Index Terms*—Analytical model, blocking probability, optical burst switching (OBS), optical networks.**

I. INTRODUCTION

OPTICAL burst switching (OBS) has been proposed as a means of accommodating bursty traffic and allowing sub-wavelength granularity in an all-optical manner. In OBS systems, the header and payload of bursts are separated by a time offset to allow for the processing time of the header (control packet) at intermediate nodes. An additional offset, which we term a *quality-of-service (QoS) offset* can also be added to increase a burst's priority and implement priority classes [1]. Furthermore, if the offset for class i bursts is longer than the offset of class j bursts *plus* the *maximum* class j burst length, then class i will be isolated from class j [2]. In this letter, we consider OBS systems that use Just-Enough-Time signaling [1] and that do not employ optical buffers or wavelength converters.

Analytical models to compute the blocking probability of each class in multi-class OBS systems have previously been proposed. In [3], an elegant recursive model that assumes complete isolation between classes is presented. A more general model that does not require class isolation is presented in [4]. Models for OBS systems with optical buffers are presented in [5] and [6].

To date, all previously proposed OBS models assume that the *conservation law* holds (i.e., the OBS system is work-conserving). If the conservation law holds for an OBS system, the overall rate of blocking is unaffected by the number of classes and the size of the QoS offsets assigned to them. However, there are a number of practical traffic scenarios that lead to nonwork-conserving OBS systems. For example, one can envision a two-class OBS network carrying best effort low-class traffic and real-time high-class traffic. Because best-effort traffic is not delay sensitive, it is desirable to have very long low-class bursts to minimize header and switching overhead. However, high-class bursts may be significantly shorter in order to satisfy the strict delay requirements of the real-time traffic that they carry. It has been shown that systems in which the lower-class burst are longer (or shorter) than higher class bursts do not satisfy the conservation law [4] and cannot be represented by currently proposed models. Thus, there is a need to develop new models that can be applied to these systems.

In this letter, we present a model that can evaluate the blocking probabilities of multi-class OBS systems that are not work-conserving. The model is also applicable to work-conserving systems and to systems with arbitrary burst-length distributions and offsets. We derive the model in Sections II and III and verify its accuracy using simulation in Section IV.

II. MODELLING THE BURST-BLOCKING PROCESS

Since OBS uses advance reservation, a burst can be blocked by another burst in a number of different ways. For example, a burst may be blocked if the contending burst overlaps its head, its tail or both, as shown in Fig. 1, where T_i is the arrival time of a given class i header, L_i is the length of a class i burst, and ω_i is the offset time between the control packet and burst for class i (includes header length and assumes that processing-time offset is negligible compared to QoS offset). We can summarize all possible cases by recognizing that a given class i burst will be blocked by a class j burst, if all of the following events occur:

1) the header of the class j burst arrives before that of the class i burst;
2) the start of the class j burst arrives before the end of the class i burst;
3) the end of the class j burst arrives after the start of the class i burst.

We denote the intersection of these three events as B_{ij}. If we define $\delta_{ij} = \omega_i - \omega_j$ as the offset difference between class i and class j, then we can write

$$\begin{aligned} P[B_{ij}] &= P[(T_j < T_i) \cap (T_j + \omega_j < T_i + \omega_i + L_i) \\ &\quad \cap (T_j + \omega_j + L_j > T_i + \omega_i)] \\ &= P[(T_j - T_i < 0) \cap (T_j - T_i < \delta_{ij} + L_i) \\ &\quad \cap (T_j - T_i > \delta_{ij} - L_j)] \\ &= P[(T_j - T_i) \in (\delta_{ij} - L_j, \min(0, L_i + \delta_{ij}))] \quad (1) \end{aligned}$$

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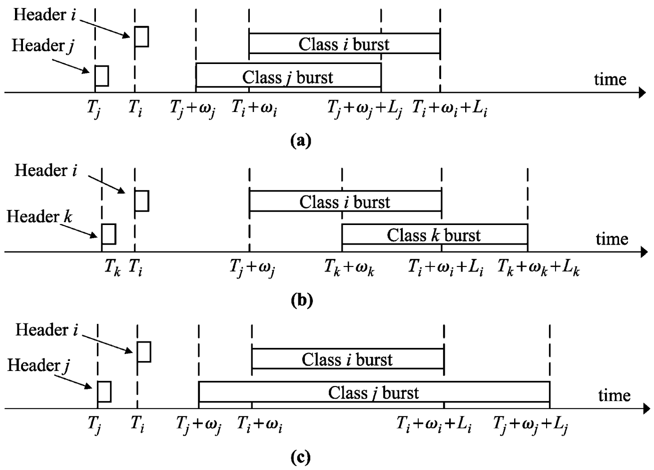

Fig. 1. Three different ways by which a class i burst can be blocked. Contending burst overlaps: (a) head of blocked burst; (b) tail of blocked burst; and (c) both head and tail of blocked burst.

If bursts in class j arrive according to a Poisson process, then we can make use of the memoryless property [7] to write

$$P[\overline{B_{ij}}] = P[\tau_j > \min(0, L_i + \delta_{ij}) - (\delta_{ij} - L_j)]$$
$$= \begin{cases} \int_0^{-\delta_{ij}} P[\tau_j > L_j + l_i] f_{L_i}(l_i) dl_i \\ + \int_{-\delta_{ij}}^{\infty} P[\tau_j > L_j - \delta_{ij}] f_{L_i}(l_i) dl_i, & \delta_{ij} < 0 \\ P[\tau_j > L_j - \delta_{ij}], & \delta_{ij} \geq 0 \end{cases}$$
$$= \begin{cases} \int_0^{-\delta_{ij}} P[\tau_j > L_j + l_i] f_{L_i}(l_i) dl_i \\ + P[\tau_j > L_j - \delta_{ij}] \left(1 - F_{L_i}(-\delta_{ij})\right), & \delta_{ij} < 0 \\ P[\tau_j > L_j - \delta_{ij}], & \delta_{ij} \geq 0 \end{cases} \tag{2}$$

where $\overline{B_{ij}}$ is the complement of B_{ij}, $f_{L_i}(l)$ and $F_{L_i}(l)$ are the probability density and distribution functions for the length of class i bursts respectively, τ_j is defined as the interarrival time of class j bursts and $P[\tau_j > L_j + l_i]$ and $P[\tau_j > L_j - \delta_{ij}]$ can be evaluated using (8) (see Appendix).

III. Multi-Class Blocking Model

In this section, we are interested in finding P_{bi} the average blocking probability of a class i burst. Without loss of generality, we assume that the network has N classes of traffic labeled $1, \ldots, N$ such that $\delta_{ij} > 0$ for all $i < j$. Thus, class 1 has the highest priority, class 2 has the second highest, etc. When a control packet arrives, it attempts to reserve a time-slot of bandwidth to accommodate its burst. We call this time-slot the *reservation window* (RW) of the burst. If one or more bursts overlap a control packet's RW, its bursts will be blocked. Thus,

$$P_{bi} = 1 - P[\text{class } i \text{ control packet finds its RW empty}]. \tag{3}$$

If a control packet finds its RW empty, then either no bursts have previously arrived in the RW, or else one or more bursts have arrived in the RW and all of them were blocked (by previously arriving burst). If we define $a_i^{(k)}$ as the event that k bursts (from any combination of classes) have arrived in the reservation window of a class i burst, we can write

$$P_{bi} = 1 - \left(P\left[a_i^{(0)}\right] + \sum_{k=1}^{\infty} P\left[a_i^{(k)} \cap \text{all } k \text{ were blocked}\right] \right)$$
$$= 1 - P\left[a_i^{(0)}\right] - \sum_{k=1}^{\infty} \Big(P\left[k \text{ bursts blocked} \,|\, a_i^{(k)}\right] \cdot P\left[a_i^{(k)}\right] \Big). \tag{4}$$

In this letter, we consider networks in which $P_{bi} \ll 1$, so blocking events are relatively rare. Thus, $P[k \text{ bursts blocked} \,|\, a_i^{(k)}] \ll P[a_i^{(0)}]$, and we can neglect the effects of all higher order terms in (4), yielding

$$P_{bi} \approxeq 1 - P\left[a_i^{(0)}\right] = 1 - P\left[\bigcap_{j=1}^{i-1} \overline{B_{ij}} \bigcap_{j=i}^{N} \overline{B_{ij}}\right] \tag{5}$$

where we have split the above intersection into two subsets, corresponding to blocking from higher-priority traffic $(j < i)$, and lower-priority traffic $(j \geq i)$.

From (2), one observes that when $i \leq j$ or equivalently when $\delta_{ij} \geq 0$, all of the $\overline{B_{ij}}$ are independent. When $\delta_{ij} \leq 0$, the $\overline{B_{ij}}$ share a dependence on L_i. Therefore they are not necessarily independent. However, after numerically testing for a possible dependence between the $\overline{B_{ij}}$ over a large range of burst lengths and offstets, we concluded that any such dependence was very weak. Thus, we make the approximation that the terms in the first intersection in (5) are independent, and write

$$P_{bi} = 1 - \prod_{j=1}^{i-1} P[\overline{B_{ij}}] \prod_{j=i}^{N} P[\overline{B_{ij}}]. \tag{6}$$

We then substitute (2) into (6), yielding

$$P_{bi} = 1 - \prod_{j=1}^{i-1} \left(\int_0^{-\delta_{ij}} P[\tau_j > L_j + l_i] f_{L_i}(l_i) dl_i + \left[1 - F_{L_i}(-\delta_{ij})\right] P[\tau_j > L_j - \delta_{ij}] \right) \cdot \prod_{j=i}^{N} P[\tau_j > L_j - \delta_{ij}]. \tag{7}$$

This is the expression for the average blocking probability of class i when there are N classes of traffic, each with arbitrary burst-length distribution and offset times.

IV. Accuracy of Model and Simulation

In order to demonstrate the accuracy of the analytical model, we simulated a two-class OBS node. For each simulation, ten million bursts were generated. Bursts arrived according to a Poisson process. The overall traffic load (sum of class 1 and class 2 traffic) was 10^{-3} Erlangs, and the ratio of class 1 (higher class) traffic to class 2 traffic was 1:5. In Fig. 2, we plot the blocking probability for each class as we vary the QoS offset of class 1 traffic, while keeping the class 2 QoS offset equal to zero. On the abscissa, we show the normalized QoS offset difference, $\hat{\delta}_{12} = \delta_{12}/\overline{L_2}$, which we have defined as the ratio of

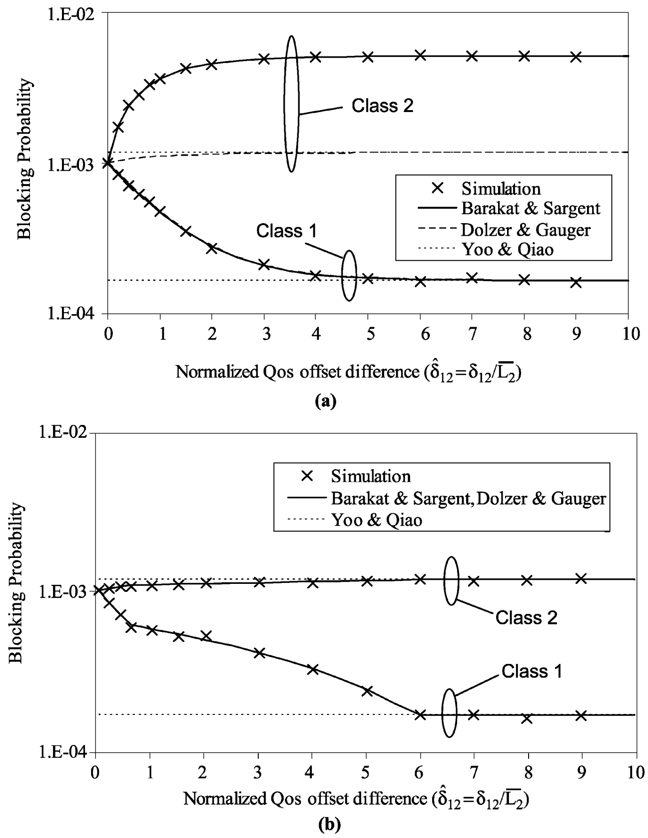

Fig. 2. Blocking Probability versus normalized QoS for OBS node with two classes. Burst lengths are exponentially distributed and system is not work-conserving in (a). Burst lengths are generally distributed and system is work-conserving in (b).

the difference of the class 1 and class 2 offsets to the mean burst length of class 2.

The present model is the first that does not assume that the OBS system is work-conserving. To demonstrate our model's accuracy for such systems, we simulated a system in which the lengths of low-class and high-class bursts were different. Bursts lengths were exponentially distributed, and the mean length of low-priority bursts was 25 times larger than that of high-priority bursts. The results are shown in Fig. 2(a).

When $\hat{\delta}_{12}$ is zero, both classes have the same blocking probability. As $\hat{\delta}_{12}$ increases and class 1 becomes more isolated from class 2, the blocking experienced by class 1 decreases and that of class 2 increases. Since burst lengths are exponentially distributed, there is no maximum burst length, so complete isolation cannot be achieved. However, for the parameters simulated, the two classes are nearly isolated for $\hat{\delta}_{12} > 7$.

The curve corresponding to the model presented in this letter matches the simulation results very closely for both classes of traffic for all values of $\hat{\delta}_{12}$ simulated. As expected, since the models in [3] and [4] assume that the system is work-conserving, their predicted blocking probabilities are significantly less accurate; their predictions for the blocking probabilities of class 2 are approximately 75% lower than the simulated values. Since the model in [3] assumes that class 1 is isolated from class 2, its computed blocking probabilities for class 1 are also inaccurate for small values of $\hat{\delta}_{12}$.

In order to demonstrate that our model is also accurate for the case of work-conserving OBS systems and the case of generally distributed burst lengths, we simulated a system in which the lengths of class 1 and class 2 bursts had the same mean and followed a general distribution. For the general burst-length distribution, 50% of bursts were the median length, 40% of bursts were short (half the medium length), and 10% of bursts were long (ten times the median length). Thus, given a mean burst length of $\overline{l_k}$ for class k, the short, medium, and long bursts had lengths of $\overline{l_k}/3.4$, $\overline{l_k}/1.7$, and $\overline{l_k}/0.17$ respectively. The results are shown in Fig. 2(b). Since the curves for our model and the model in [4] were found to be identical, they are represented by a single curve.

Once again, because the model in [3] assumes complete isolation between classes, its computed blocking probabilities are only accurate when the offset difference between class 1 and class 2 is larger than the maximum burst size of class 2 (i.e., when $\hat{\delta}_{12} \geq 0.17^{-1}$). The outputs of both the model presented in this letter and the model presented in [4] agree closely with the simulation for all values of $\hat{\delta}_{12}$.

V. CONCLUSION

In this letter, we derived an analytical model that evaluates the blocking probability for each class in a multi-class OBS system with arbitrary burst-length distributions and arbitrary offsets. The model can be applied to both work-conserving systems and nonwork-conserving systems. Using simulation, we showed that our model computes the average blocking probability experienced by each classes of traffic for a nonwork-conserving system more accurately than previously proposed models. The model was also shown to be very accurate for work-conserving systems.

APPENDIX

If X and Y are independent, nonnegative random variables with probability density functions $f_X(x)$ and $f_Y(y)$, and c is a constant

$$P[X > Y + c] = \begin{cases} \int_0^\infty \int_0^{x-c} f_X(x) f_Y(y) dy\, dx, & c < 0 \\ \int_0^\infty \int_{y+c}^\infty f_X(x) f_Y(y) dx\, dy, & c \geq 0. \end{cases} \tag{8}$$

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